

Comparing Linear Approximations to Calculator Computations

In lecture, we explored linear approximations to common functions at the point $x = 0$. In this worked example, we use the approximations to calculate values of the sine function near $x = 0$ and compare the answers to those on a scientific calculator.

Find the linear approximation to $\sin(x)$ at the point $x = 0$ and use your answer to approximate the values of $\sin(.01)$, $\sin(.1)$ and $\sin(1)$. Check your answer on a calculator.

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$$f(x) \approx f(x_0) + f'(x_0)(x-x_0) \text{ when } x \approx x_0$$

$$f(x) = \sin x, x_0 = 0$$

$$\begin{aligned} \Rightarrow f(x) &\approx \sin 0 + \cos 0 (x-0) \\ &= 0 + 1(x) \\ &= x \end{aligned}$$

$$\therefore \sin x \approx x \text{ when } x \approx 0.$$

$$\sin(0.01) = 0.01$$

$$\sin(0.1) = 0.1$$

$$\sin(1) = 1$$

Calculator:

$$\sin(0.01) = 0.00999983$$

$$\sin(0.1) = 0.09988234$$

$$\sin(1) = 0.84147098$$

$$\Delta f_{0.01} > 0.0001$$

$$\Delta f_1 > 0.158$$